Radiative ρ-Meson Decay*

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The radiative decays of the charged and neutral ρ meson are discussed assuming that the decay vertices are dominated by the low-mass $\pi\pi$ and $\omega\pi$ intermediate states. The transitions $\rho\rightarrow\pi+\gamma$, $\rho^0\rightarrow\pi^++\pi^-+\gamma$, $\rho^0\rightarrow\pi^0+\pi^0+\gamma$, $\rho^{+,-}\rightarrow\pi^{+,-}+\pi^0+\gamma$ and $\rho^0\rightarrow\eta+\gamma$ are considered in detail. The probability for the emission of a photon with energy exceeding 15 MeV in the process $\rho^0\rightarrow\pi^++\pi^-+\gamma$ relative to the 2π decay is found to be 1.7×10^{-2}, indicating that the $(2\pi+\gamma)$ decay width is comparable or greater than the $(\pi+\gamma)$ one. The internal bremsstrahlung and direct emission processes are included in the calculation of the various partial widths.

1. INTRODUCTION

THE 750-MeV isovector ρ meson has been observed only through its two-pion main decay mode[1] in the various charge states. Its attributed quantum numbers are 1^{-+} in the customary notation (spin, parity, G parity).

The appreciable background of two-pion nonresonant states which accompanies the ρ-meson production in most of the reported experiments, and its large width (100–120 MeV), have precluded to date a search for other less frequent decays of the ρ (e.g., the electromagnetic decays). Typical of the present experimental status is, for example, the recently reported[2] ratio of $\rho^0\rightarrow$ neutrals (presumably mostly $\pi^0+\gamma$, as a $2\pi^0$ decay is strictly forbidden)$/\rho^0\rightarrow\pi^++\pi^-\leq(6\pm35)\%$. However, future adequate experiments with ρ's produced at higher energies will probably give much cleaner samples[3] which could be used to study the unfrequent decays. The additional decays will throw light on both electromagnetic and strong interactions of the ρ meson.

We intend to discuss in this work the first-order (in the "fine-structure" constant $\alpha=1/137$) electromagnetic decays of the ρ meson. The most frequent ones in this class are expected to be the $(\pi+\gamma)$, $(2\pi+\gamma)$, and probably $(\eta+\gamma)$ decays.

Generally, a radiative decay of particle A, namely, $A\rightarrow\sum_i B_i+\gamma$, can possibly occur through two distinct mechanisms: (a) emission of a photon by an ingoing or outgoing charged particle participating in the process $A\rightarrow\sum_i B_i$ (internal bremsstrahlung), and (b) "direct" emission process which reflects the internal structure of the interactions of the decaying particle. The $\rho\rightarrow\pi+\gamma$ can occur only as a "direct" decay, while the $\rho\rightarrow2\pi+\gamma$ process has also an "internal bremsstrahlung" part. Hence, no obvious argument can be given *a priori* as to which of the two is more frequent. The $(3\pi+\gamma)$ decay has only a direct part; therefore, we expect its width to be much smaller than the partial widths for the $(\pi+\gamma)$ and $(2\pi+\gamma)$ decays and we will not discuss it further in this work.[4,5]

Clearly, the estimates we are going to make of the direct parts of the radiative decays will be model-dependent. The use of a specific model employed in this work is suggested by its ability of giving reasonable results for the ω decays, as discussed in the next section. Unless the ρ meson has some unexpected features (e.g., large anomalous magnetic moment), the calculation presented here of the "internal bremsstrahlung" part in the $(2\pi+\gamma)$ decay, which mainly involves the rules of the quantum electrodynamics, should be quite reliable.

2. METHOD

Gell-Mann and Zachariasen[6] have proposed a "pole approximation" method to calculate the $\rho\rightarrow\pi+\gamma$ and $\omega\rightarrow\pi+\gamma$ decays. Namely, they assumed in the sense of dispersion theory that the dominant intermediate states and $(\omega\pi)$ and $(\rho\pi)$, respectively, followed by the transitions $\omega\rightarrow\gamma$ and $\rho\rightarrow\gamma$. Although we know that both ω and ρ interact with nucleons, since the above-mentioned intermediate states are appreciably lighter than a nucleon-antinucleon pair, it is reasonable to expect also in a perturbation theory that the nucleon states give only a small correction, especially for a strong enough coupling constant $f_{\rho\omega\pi}$. This model was used to estimate the[7] $(\omega\rightarrow\pi+\gamma)/(\omega\rightarrow3\pi)$ and[5]

* Research supported by the National Science Foundation.

[1] D. Stonehill, C. Baltay, H. Courant, W. Flickinger E. C., Fowler, H. Kraybill, J. Sandweiss, J. Sanford, and H. Taft, Phys. Rev. Letters **6**, 624 (1961); A. R. Erwin, R. March, W. D. Walker, and E. West, *ibid.* **6**, 628 (1961); E. Pickup, D. K. Robinson, and E. O. Salant, *ibid.* **7**, 192 (1961); C. Alff, D. Berley, D. Colley, N. Gelfand, U. Nauenberg, D. Miller, J. Schultz, J. Steinberger, T. H. Tan, H. Brugger, P. Kramer, and R. Plano, *ibid.* **9**, 322 (1962).

[2] R. Strand, R. Kraemer, M. Meer, M. Nussbaum, A. Pevsner, C. Richardson, T. Toohig, M. Block, S. Orenstein, and T. Fields, in *Proceedings of the 1962 Annual International Conference on High-Energy Physics at CERN* (CERN, Geneva, Switzerland, 1962).

[3] D. O. Caldwell, E. Bleuler, B. Elsner, L. W. Jones, and N. Zacharov, Phys. Letters **2**, 253 (1962).

[4] The $\rho\rightarrow3\pi+\gamma$ decay can be estimated by assuming the following mechanism: $\rho\rightarrow(\omega)+\pi\rightarrow(2\pi+\gamma)+\pi$. Within this approximation, since the width of ω for $2\pi+\gamma$ decay is significantly smaller than for $\pi+\gamma$ decay (see reference 5), clearly $\Gamma_\rho(3\pi+\gamma)\ll\Gamma_\rho(2\pi+\gamma)$ ("direct"). The decay $\rho\rightarrow\eta+\gamma$ will appear in 25% of the cases as a $\rho\rightarrow3\pi+\gamma$ decay, but it can be identified through its monoenergetic γ ray.

[5] P. Singer, Phys. Rev. **128**, 2789 (1962).

[6] M. Gell-Mann and F. Zachariasen, Phys. Rev. **125**, 953 (1961).

[7] M. Gell-Mann, D. Sharp, and W. G. Wagner, Phys. Rev. Letters **8**, 261 (1962).

$(\omega \to 2\pi+\gamma)/(\omega \to 3\pi)$ ratios with apparent fair success, keeping in mind the still scarce experimental information. We propose in this article to use the same vertex to estimate the $\rho \to 2\pi+\gamma$ direct decay (in particular, that part of this decay which cannot be obtained by gauge invariance, as it will be explained in the next section), assuming it to proceed $\rho \to (\omega)+\pi \to (\pi+\gamma)+\pi$. This method, combined with unitary symmetry[8] and the assumption that the ρ meson dominates the electric pion form factor, enables us to obtain the different electromagnetic and strong decays of ω and ρ expressed in terms of two coupling constants, $f_{\rho\omega\pi}$ and $f_{\rho\pi\pi}$[9] (besides, of course, the electromagnetic coupling constant). $f_{\rho\omega\pi}$ is still unknown although measurable, while $f_{\rho\pi\pi}$ is deducible from the measured width of the 2π decay of ρ. Remembering our inability of handling a Lagrangian approach to this problem with the strongly interacting nucleons included, the method described, if the measured rates will continue to prove it reliable, should be considered as an interesting and valuable substitute.

It has been tried[6,7] to estimate the $f_{\rho\omega\pi}$ coupling constant from the measured width of the π^0 decay, supposedly proceeding predominantly via $\pi^0 \to \rho+\omega \to \gamma+\gamma$. However, in this case the mass of the intermediate states considered is comparable to that of possible nucleon-antinucleon intermediate states and, therefore, cancellations which could change considerably the outlined estimate are likely to occur.[10] Hence, the $f_{\rho\omega\pi}$ coupling constant still awaits a more direct measurement. But it should be kept in mind that the possible inadequacy of the "pole approximation" in the π^0 decay does not invalidate the use of the method in the ρ and ω decays. In this article, we do not include any form factor dependence for the $\rho\omega\pi$ vertex.

In discussing the first-order electromagnetic decays of ρ, one should mention also the possible $\rho^0 \to \eta+\gamma$ process (the very probable quantum numbers of η are 0^{-+}). Similar to the philosophy of the predominance of the $\rho\omega\pi$ vertex in the pionic radiative decays of ρ and ω, we assume the $\rho\rho\eta$ vertex to be responsible for the η radiative decay of the ρ meson. Previously, this vertex was used to estimate the 2γ and $2\pi+\gamma$ decays of the η.[7,11] In that case, an argument similar to the one mentioned in connection with the π^0 decay could be invoked. Nevertheless, the use of the $\rho\rho\eta$ vertex in the $\rho \to \eta+\gamma$ process is obviously more satisfactory.

To summarize, we shall discuss the electromagnetic decays of ρ by conjecturing the dominance of low-mass intermediate states. The states included are 2π and 4π, and we assume for the latter to be practically an $\omega\pi$ state.

[8] M. Gell-Mann, Phys. Rev. **125**, 1067 (1962).

[9] In making this statement we imply $f_\omega = f_\rho$ from unitary symmetry and $f_\rho = f_{\rho\pi\pi}$ from the ρ dominance of the pion form factor. See references 6–8 for more detailed discussion.

[10] D. A. Geffen, Phys. Rev. **128**, 374 (1962).

[11] L. M. Brown and P. Singer, Phys. Rev. Letters **8**, 460 (1962).

3. DECAY OF NEUTRAL ρ MESON

For completeness, we shall briefly discuss the $\rho \to \pi+\gamma$ decay, although it was previously treated[6] along the same lines. If one assumes that the decay is dominated by $\rho^0 \to \omega+\pi^0$ followed by $\omega \to \gamma$ one obtains,[12]

$$\Gamma_{\rho^0}(\pi^0\gamma) = \alpha(f_{\rho\omega\pi}^2/4\pi)(f_\omega^2/4\pi)^{-1} \times (m_\rho^2 - m_\pi^2)^3(72)^{-1} m_\rho^{-3} m_\pi^{-2}. \quad (1)$$

In obtaining (1) one uses for the $\rho\omega\pi$ vertex the expression

$$(f_{\rho\omega\pi}/m_\pi)\epsilon_{\alpha\beta\gamma\delta}p_\alpha^{(\rho)}\epsilon_\beta^{(\rho)}p_\gamma^{(\omega)}\epsilon_\delta^{(\omega)}, \quad (2)$$

where $\epsilon_{\alpha\beta\gamma\delta}$ is the 4th-order completely antisymmetric unit tensor, the p's are the energy-momentum 4-vectors and the ϵ's the polarization vectors of the indicated particles. At the vertex of the $\omega \to \gamma$ transition we insert[6,8] $(em_\omega^2/\sqrt{3}f_\omega)$, while for the $\rho^0 \to \gamma$ transition one writes (em_ρ^2/f_ρ). We estimate the value of f_ω as explained in reference 9.

Different approaches for estimating the $\Gamma_\rho(\pi\gamma)/\Gamma_\rho(2\pi)$ ratio have appeared in the literature.[13,14] For example, one could take for the pertinent electromagnetic and strong interactions the simplest form compatible with Lorentz and gauge invariance, without assuming any detailed structure. Then the interaction Lagrangian densities describing the respective decays are

$$L(\rho^+ \to \pi^+\gamma) = \frac{ieG}{2m}\left(\rho_\mu{}^+ \frac{\partial\pi^+}{\partial x_\nu} F_{\sigma\tau}\right)\epsilon_{\mu\nu\sigma\tau} + \text{H.c.}$$

and

$$L(\rho \to 2\pi) = \frac{i}{2} G\rho_\mu{}^a(\pi_b \overleftrightarrow{\partial}_\mu \pi_c)\epsilon_{abc}$$

in an obvious notation (a, b, c are isotopic spin indices). One then obtains in first-order perturbation theory $\Gamma_\rho(\pi\gamma)/\Gamma_\rho(2\pi) = 5.2\%$.[14] The strong coupling is eliminated from the ratio in this approach. In the model we use, the $\rho \to \pi+\gamma$ partial width is related to the ω width for 3π decay through the $\rho\omega\pi$ vertex. Numerical figures, which could differ appreciably from the "perturbational approach" are presented at the end of this section.

Now we turn to the $\rho \to \eta+\gamma$ decay. By using an expression similar to (2) for the $\rho\rho\eta$ vertex (with ρ replacing ω) one obtains

$$\Gamma_{\rho^0}(\eta\gamma) = \alpha(f_{\rho\rho\eta}^2/4\pi)(f_\rho^2/4\pi)^{-1} \times (m_\rho^2 - m_\eta^2)^3(24)^{-1} m_\rho^{-3} m_\pi^{-2}. \quad (3)$$

If we conjecture approximate unitary symmetry,

[12] We use rationalized natural units such that $\hbar = c = 1$ and $e^2/4\pi = 1/137$.

[13] G. Feinberg, Phys. Rev. Letters **8**, 151 (1962).

[14] C. H. Chan, Proc. Phys. Soc. (London) **80**, 39 (1962).

namely, $f_{\rho\rho\eta}\simeq f_{\rho\omega\pi}$ and $f_\rho\simeq f_\omega$, we get

$$\Gamma_{\rho^0}(\pi^0\gamma)/\Gamma_{\rho^0}(\eta\gamma) = (m_\rho^2-m_\pi^2)^3/3(m_\rho^2-m_\eta^2)^3=3.3. \quad (4)$$

As the masses of the π and η are quite different, indicating a break in the unitary symmetry, the same may be true for the respective coupling constants. Then, the estimate (4) will change by the appropriate factor. The factor $\frac{1}{3}$ in (4) is due to the fact that the $(\pi+\gamma)$ transition proceeds through the isoscalar part of the electromagnetic current, while the $(\eta+\gamma)$ transition involves the isovector part.[15] The γ energies in these decays are 363 MeV for the $\rho\to\pi+\gamma$ decay and 173 MeV for the $\rho^0\to\eta+\gamma$ transition.

The decay $\rho^0\to 2\pi+\gamma$ can proceed only through electric transitions because of parity and charge conjugation invariance (the charge conjugation quantum number for the ρ^0 meson is -1). Therefore, the lowest allowed final state is an S state for the pions, realized by an electric dipole transition. The $\rho^0\to\pi^++\pi^-+\gamma$ process will have an "internal bremsstrahlung" part contributed by the emission of photons by the outgoing charged pions from the decay $\rho^0\to\pi^++\pi^-$, and a possible "direct" part. The $\rho^0\to 2\pi^0+\gamma$ decay is possible only as a direct process, and will be visualized in our model as proceeding $\rho^0\to(\omega)+\pi^0\to(\pi^0+\gamma)+\pi^0$. This is the only final state which can be obtained in the ρ^0 decay from the dominant intermediate $\omega\pi$ state. Hence, in our picture, the $\rho^0\to\pi^++\pi^-+\gamma$ and the $\rho^0\to 2\pi^0+\gamma$ occur through completely different channels.

In order to calculate the $\rho^0\to\pi^++\pi^-+\gamma$ decay, the knowledge of the $\rho^0\to\pi^++\pi^-$ vertex is needed. As is well known, the ρ meson is supposed to be the vector meson coupled to the conserved isotopic spin current.[16] An analysis of the existing experimental data[17] shows good evidence for the universality of the ρ meson coupling to pions and nucleons, giving $f_{\rho\pi\pi}^2/4\pi \simeq f_{\rho NN}^2/4\pi\simeq 2$. Accordingly, we take for the $\rho\to\pi_1+\pi_2$ vertex the expression

$$f_{\rho\pi\pi}\epsilon^{(\rho)}\cdot(p_1-p_2), \quad (5)$$

and we assume in the following no form factor dependence for the $\rho\leftrightarrow 2\pi$ vertex. The main contribution to the photon spectrum comes from the low-energy photons; therefore, no strong form-factor effects for the off-the-mass-shell expression (5) is to be expected.

The matrix element for the emission of a photon with 4-momentum k, as picturized in Figs. 1(a) and 1(b), is given[18] by

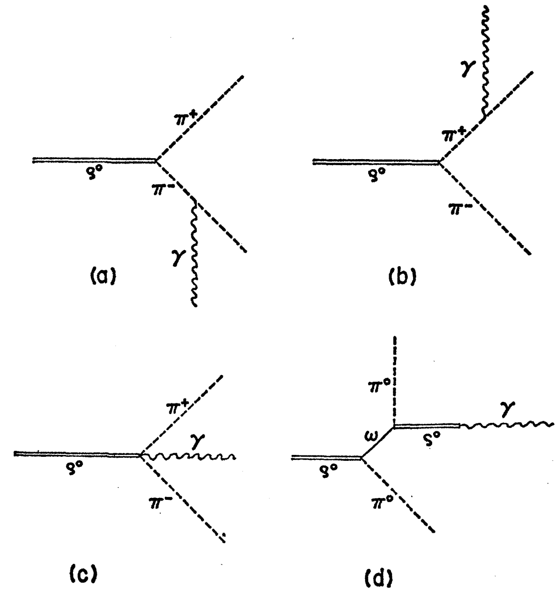

FIG. 1. Diagrams for decay $\rho^0\to 2\pi+\gamma$.

$$M_1=ef_{\rho\pi\pi}\epsilon^{(\rho)}\cdot(p^{(+)}-p^{(-)})\left(\frac{p^{(+)}\cdot\epsilon^{(\gamma)}}{p^{(+)}\cdot k}-\frac{p^{(-)}\cdot\epsilon^{(\gamma)}}{p^{(-)}\cdot k}\right) +ef_{\rho\pi\pi}\epsilon^{(\rho)}\cdot k\left(\frac{p^{(+)}\cdot\epsilon^{(\gamma)}}{p^{(+)}\cdot k}+\frac{p^{(-)}\cdot\epsilon^{(\gamma)}}{p^{(-)}\cdot k}\right). \quad (6)$$

The first term in (6) is the well-known bremsstrahlung term which would appear alone if the $\rho\leftrightarrow 2\pi$ transition were structure independent. The second term appears because of the special form (5) of this transition. The expression (6) is not gauge invariant and needs to be supplemented by the "direct emission" term [Fig. 1(c)], which can be obtained by replacing in (5) $p^{(+)}\to p^{(+)}-e\epsilon^{(\gamma)}$ and $p^{(-)}\to p^{(-)}+e\epsilon^{(\gamma)}$. This additional term has the form

$$M_2=-2ef_{\rho\pi\pi}\epsilon^{(\rho)}\cdot\epsilon^{(\gamma)}. \quad (7)$$

It can be easily checked that M_1+M_2 is gauge invariant. We would like to emphasize that generally, not all the direct terms can be deduced from gauge invariance, as outlined above. A general term, which vanishes as $k\to 0$ and is Lorentz and gauge invariant, can be added to $M=M_1+M_2$ to represent additional direct processes. However, in our case, as we assume that the vertices $\rho\pi\pi$ and $\rho\omega\pi$ are sufficient in calculating the ρ decays, and no $\pi^+\pi^-\gamma$ final state can be obtained in first order from the $\rho\omega\pi$ vertex, (6) and (7) contain all the possible processes leading to $\rho^0\to\pi^++\pi^-+\gamma$.

It is interesting to remark that in the radiative decay

[15] In the case of the ω decays to $(\pi^0+\gamma)$ or $(\eta+\gamma)$ this factor operates in the opposite direction, so that we obtain $(\Gamma_\omega(\pi^0\gamma)/\Gamma_\omega(\eta\gamma))=3(m_\omega^2-m_\pi^2)^3/(m_\omega^2-m_\eta^2)^3=24$. This reduces the $(\eta+\gamma)$ mode to only about 4% of the $(\pi+\gamma)$ mode, compared to the 35% expected in the ρ case.

[16] C. N. Yang and R. L. Mills, Phys. Rev. **96**, 191 (1954); J. J. Sakurai, Ann. Phys. (N. Y.) **11**, 1 (1960).

[17] J. J. Sakurai, in Proceedings of the International School of Physics "Enrico Fermi," Varenna, Como, Italy, 1962 (to be published).

[18] In our notation and chosen metric $A\cdot B=A_0B_0-\mathbf{A}\cdot\mathbf{B}$.

$K \to 2\pi+\gamma$, because of cancellations, only an expression parallel to the first term in (6) remains in that part of the matrix element for the radiative decay which is calculated by relating it to the matrix element for $K \to 2\pi$.[19]

The partial width for the decay $\rho^0 \to \pi^+ + \pi^- + \gamma$ is now given in terms of (6) and (7) by

$$\Gamma_{\rho^0}(\pi^+\pi^-\gamma) = \frac{1}{2m_\rho}\int (2\pi)^4\delta(p^{(\rho)}-p^{(+)}-p^{(-)}-k)(\sum|M_1+M_2|^2) \times \frac{d^3p^{(+)}}{(2\pi)^3 2p_0^{(+)}}\frac{d^3p^{(-)}}{(2\pi)^3 2p_0^{(-)}}\frac{d^3k}{(2\pi)^3 2k_0}. \quad (8)$$

$\sum$ denotes average over the initial ρ polarization and sum over the photon polarization states. The integration over $p^{(+)}$ and $p^{(-)}$ is most easily carried out by using a four-dimensional notation,[20] with the phase-space volumes of $p^{(+)}$ and $p^{(-)}$ given by

$$\frac{d^3p^{(+)}}{p_0^{(+)}}\frac{d^3p^{(-)}}{p_0^{(-)}} = 4\delta(p^{(+)2}-m_\pi^2)\delta(p^{(-)2}-m_\pi^2)d^4p^{(+)}d^4p^{(-)} = \delta(Q\cdot R)\delta(Q^2+R^2-4m_\pi^2)d^4Q d^4R, \quad (9)$$

where $Q=p^{(+)}+p^{(-)}$ and $R=p^{(+)}-p^{(-)}$. We obtain finally for the probability of emission of a photon with energy exceeding k in the process $\rho^0 \to \pi^+ + \pi^- + \gamma$

$$\Gamma_{\rho^0}(\pi^+\pi^-\gamma) = 2\alpha(f_{\rho\pi\pi}^2/4\pi)(3\pi m_\rho)^{-1}\int_k^{k_m} I(k)\frac{dk}{k}, \quad (10)$$

where k_m is the maximum possible γ energy in this process given by

$$k_m = (m_\rho/2) - (2m_\pi^2/m_\rho) = 323 \text{ MeV}.$$

For the γ-energy spectrum $I(k)$ one has

$$I(k) = \left[k_m\left(\frac{m_\rho}{2}-k\right)+\frac{m_\pi^2}{m_\rho}(4k-k_m)\right]\ln\frac{1+\xi}{1-\xi} - \xi\left[\left(\frac{m_\rho}{2}-k\right)(k_m+4k)-k^2\right], \quad (12)$$

where

$$\xi = \left[(k_m-k)\bigg/\left(\frac{m_\rho}{2}-k\right)\right]^{1/2}. \quad (13)$$

The partial width for the decay $\rho^0 \to \pi^+ + \pi^-$ using (5) comes out to be

$$\Gamma_{\rho^0}(\pi^+\pi^-) = (f_{\rho\pi\pi}^2/4\pi)(12)^{-1}(m_\rho)^{-2}(m_\rho^2-4m_\pi^2)^{3/2}, \quad (14)$$

so that the relative probability for emitting a photon with energy greater than k is

$$R_k = \frac{\Gamma_{\rho^0}(\pi^+\pi^-\gamma)}{\Gamma_{\rho^0}(\pi^+\pi^-)} = 8\alpha m_\rho(\pi)^{-1}(m_\rho^2-4m_\pi^2)^{-3/2} \times \int_k^{k_m} I(k)\frac{dk}{k}. \quad (15)$$

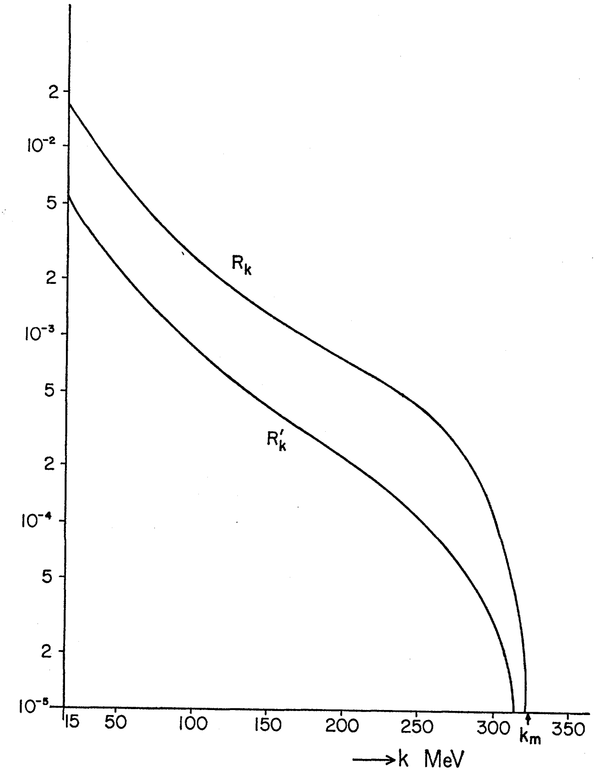

FIG. 2. The relative probabilities $R_k=[\Gamma_{\rho^0}(\pi^+\pi^-\gamma)/\Gamma_{\rho^0}(\pi^+\pi^-)]$ and $R_k'=[\Gamma_{\rho^{+,-}}(\pi^{+,-}\pi^0\gamma)/\Gamma_{\rho^{+,-}}(\pi^{+,-}\pi^0)]$ for emission of a photon with energy exceeding k MeV.

R_k is plotted in Fig. 2 as a function of k. For convenience, a few values of R_k are tabulated in Table I.

A γ energy of 15–30 MeV is a reasonable lower limit of the experimental detection ability. One notices an unusually high bremsstrahlung radiative decay probability, compared to other known processes like[20] $K \to 3\pi+\gamma$ or[21] $K \to 2\pi+\gamma$. This is mainly due to the large phase space available in ρ decay. Moreover, because of the contributions of the "direct" term (7) and the second term in (6), the slope of the R_k curve is somewhat different from the one encountered in the K radiative decays. Obviously, the contribution of these terms is mainly in the high-energy range of the spectrum. To give an idea on the magnitude of the "direct" term, if one takes only the expression (7) to calculate

[19] H. Chew, Phys. Rev. **123**, 377 (1961).
[20] R. H. Dalitz, Phys. Rev. **99**, 915 (1955).
[21] J. D. Good, Phys. Rev. **113**, 352 (1959).

TABLE I. The relative probabilities $R_k=[\Gamma_{\rho^0}(\pi^+\pi^-\gamma)/\Gamma_{\rho^0}(\pi^+\pi^-)]$ and $R_k'=[\Gamma_{\rho^{+,-}}(\pi^{+,-}\pi^0\gamma)/\Gamma_{\rho^{+,-}}(\pi^{+,-}\pi^0)]$[a] for emitting a photon with energy greater than k.

k (MeV)	15	30	45	60	105	165	225	285
R_k	1.7×10^{-2}	1.0×10^{-2}	7.2×10^{-3}	5.3×10^{-3}	2.4×10^{-3}	1.1×10^{-3}	5.8×10^{-4}	2.1×10^{-4}
R_k'	5.6×10^{-3}	3.5×10^{-3}	2.4×10^{-3}	1.7×10^{-3}	7.8×10^{-4}	3.3×10^{-4}	1.6×10^{-4}	4.7×10^{-5}

[a] R_k' does not include the contribution of the $\omega\pi$ intermediate states to the $\rho^{+,-}\rightarrow\pi^{+,-}+\pi^0+\gamma$ transition.

the decay under discussion, one obtains

$$\Gamma'=2\alpha(f_{\rho\pi\pi}{}^2/4\pi)(3m_\rho\pi)^{-1}\int_0^{k_m}k\xi dk,$$

which gives for the ratio R a value of 1.6×10^{-3}. The k energy spectrum for this term starts at 0 and has a maximum at about $k=255$ MeV.

The calculation of the partial width for the decay $\rho^0\rightarrow2\pi^0+\gamma$, proceeding through an $\omega\pi$ intermediate state [Fig. 1(d)] is identical to the calculation of the $\omega\rightarrow2\pi+\gamma$ decay described in reference 5. One obtains

$$\Gamma_{\rho^0}(\pi^0\pi^0\gamma)=(f_{\rho\omega\pi}{}^2/4\pi)(f_{\omega\pi\gamma}{}^2/4\pi)(m_\rho/m_\pi)^4m_\rho(48\pi)^{-1}\times\int_0^{x_m}F(x)dx, \quad (16)$$

where $x=k/m_\rho$ and $x_m=k_m/m_\rho$. The photon energy spectrum is given by

$$F(k)=A(k)\beta(k)+B(k)\ln\frac{\alpha(k)+\beta(k)}{\alpha(k)-\beta(k)}, \quad (17)$$

where

$$\alpha(k)=m_\omega{}^2-m_\pi{}^2-m_\rho k, \quad (18a)$$

$$\beta(k)=m_\rho k\left[(k_m-k)\bigg/\left(\frac{m_\rho}{2}-k\right)\right]^{1/2}, \quad (18b)$$

and $A(k)$ and $B(k)$ are the expressions [10(a)] and [10(b)] of reference 5 with m_ρ and m_ω interchanged. The γ energy spectrum is practically identical to the one from the decay $\omega\rightarrow2\pi^0+\gamma$ given also in reference 5. $f_{\omega\pi\gamma}$ is the effective coupling for the $\omega\rightarrow\pi+\gamma$ decay, and if we take it as proceeding through a $\rho\pi$ intermediate state as discussed in Sec. 2, one obtains after the integration over $F(x)$

$$\Gamma_{\rho^0}(\pi^0\pi^0\gamma)=\alpha(f_{\rho\omega\pi}{}^2/4\pi)^2(f_\rho{}^2/4\pi)^{-1}\times(m_\rho/m_\pi)^4m_\rho(48\pi)^{-1}(1.05\times10^{-3}). \quad (19)$$

In Table II we give the ratios for $\Gamma_\rho(\pi\gamma)/\Gamma_\rho(\pi\pi)$ and $\Gamma_{\rho^0}(\pi^0\pi^0\gamma)/\Gamma_{\rho^0}(\pi^+\pi^-)$ as a function of the $f_{\rho\omega\pi}$ coupling constant. For $f_\rho=f_\omega=f_{\rho\pi\pi}$ we use a value of 2, corresponding to a ρ width of 100 MeV. The values of $f_{\rho\omega\pi}$ were so chosen as to correspond to the maximum experimental limit put on the ω width ($\simeq$20 MeV), to a preliminary value for the ratio $\Gamma_\rho(\pi\gamma)/\Gamma_\rho(2\pi)$ measured indirectly by evaluating the $\rho\pi\gamma$ vertex from ρ photoproduction (0.6%) and to the value of $f_{\rho\omega\pi}$ calculated by assuming the π^0 decay proceeding predominantly through the $\rho\omega\pi$ vertex (2.2×10^{-2} for a π^0 decay width of $\Gamma_{\pi^0}(2\gamma)=5\times10^{15}$ sec^{-1}).

Comparing Tables I and II one sees that the decay $\rho^0\rightarrow\pi^++\pi^-+\gamma$ (for $k>15$ MeV) is more frequent than $\rho^0\rightarrow\pi^0+\gamma$, even if $f_{\rho\omega\pi}{}^2/4\pi$ will prove to be of the order of unity. Of course, it may be that the $\rho\rightarrow\pi+\gamma$ process is not adequately described by the "pole approximation." But even then, as it is not expected to be more frequent than a few percent of the main decay mode, the $\pi^+\pi^-\gamma$ mode is comparable with it.

TABLE II. Decay widths of various "direct" radiative ρ decays occuring through the $\rho\omega\pi$ vertex, for different values of $f_{\rho\omega\pi}{}^2/4\pi$. The corresponding ω width is also given.

$f_{\rho\omega\pi}{}^2/4\pi$	$\Gamma_\rho(\pi\gamma)/\Gamma_\rho(2\pi)$	$\frac{\Gamma_{\rho^0}(\pi^0\pi^0\gamma)}{\Gamma_{\rho^0}(2\pi)}$	$\frac{\Gamma_{\rho^{+,-}}(\pi^+\pi^-\gamma)}{\Gamma_{\rho^{+,-}}(2\pi)}$	$\Gamma_\omega(3\pi)$ (MeV)
2.2×10^{-2}	2.2×10^{-4}	8.8×10^{-8}	8.2×10^{-7}	0.39
0.60	6.0×10^{-3}	6.5×10^{-5}	6.1×10^{-4}	11
1.1	1.1×10^{-2}	2.2×10^{-4}	2.0×10^{-3}	20

4. DECAY OF CHARGED ρ MESON

The discussion of the radiative decays of the charged ρ meson, although involving some additional features, is generally parallel to the exposition in Sec. 3.

Assuming the existence of the well-established "new" mesons only (ρ,ω,π,η), the decays to be considered are $\rho^{+,-}\rightarrow\pi^{+,-}+\gamma$ and $\rho^{+,-}\rightarrow\pi^{+,-}+\pi^0+\gamma$. It can be easily shown[22] that to first order in α,

$$\Gamma_{\rho^0}(\pi^0\gamma)=\Gamma_{\rho^{+,-}}(\pi^{+,-}\gamma), \quad (20)$$

a slight difference being possible due to the phase-space correction for $\rho^{+,-}-\rho^0$ and $\pi^{+,-}-\pi^0$ mass difference.

In discussing the $(2\pi+\gamma)$ decay, one has to remember that the initial and final states are no longer eigenstates of the charge conjugation operator. This would allow magnetic transitions in the charged ρ decay, so that in addition to the even angular momentum pion states encountered in the ρ^0 decay, odd angular momentum states are also permitted.

The $\pi^{+,-}\pi^0\gamma$ state is obtainable either as an "internal bremsstrahlung" process (and the related "direct" one required by gauge invariance) which follows a $\rho^{+,-}\rightarrow\pi^{+,-}+\pi^0$ transition, or as a direct decay proceeding through the $\rho\omega\pi$ vertex. Hence, in the decay of the

[22] G. Feinberg and A. Pais, Phys. Rev. Letters **9**, 45 (1962).

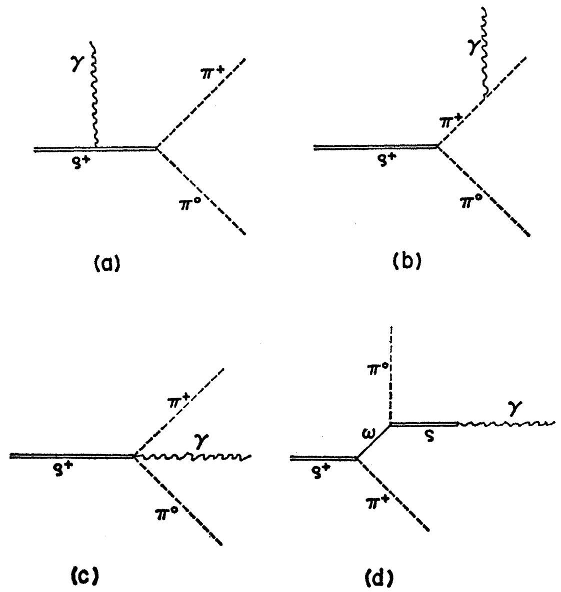

FIG. 3. Diagrams for decay $\rho^+ \to \pi^+ + \pi^0 + \gamma$.

charged ρ to $2\pi+\gamma$, the contributions related to the $\rho\pi\pi$ and $\rho\omega\pi$ vertices will interfere, while in the decay of the neutral ρ they lead to different final states.

In the following, we shall calculate separately these two contributions to the $\rho^{+,-} \to \pi^{+,-} + \pi^0 + \gamma$ decay. The calculation of the interference term is quite tedious, though straightforward. As long as our lack of knowledge of the ω width prevents us from knowing its relative importance, we feel the effort would not be justified.

For the matrix element of the transition $\rho^+ \to \pi^+ + \pi^0 + \gamma$, represented by the Feynman diagrams 3(a) and 3(b) we obtain

$$M_1^{(+)} = e f_{\rho\pi\pi} \epsilon^{(\rho)} \cdot (p^{(+)} - p^{(0)}) \left(\frac{p^{(+)} \cdot \epsilon^{(\gamma)}}{p^{(+)} \cdot k} - \frac{p^{(\rho)} \cdot \epsilon^{(\gamma)}}{p^{(\rho)} \cdot k} \right) + e f_{\rho\pi\pi} \epsilon^{(\rho)} \cdot k \frac{p^{(+)} \cdot \epsilon^{(\gamma)}}{p^{(+)} \cdot k} + \frac{e f_{\rho\pi\pi}}{2 p^{(\rho)} \cdot k} \times [(\epsilon^{(\rho)} \cdot \epsilon^{(\gamma)}) k \cdot (p^{(+)} - p^{(0)}) - (\epsilon^{(\rho)} \cdot k) \epsilon^{(\gamma)} \cdot (p^{(+)} - p^{(0)})]. \quad (21)$$

The direct term, represented in Fig. 3(c), which will complement $M_1^{(+)}$ to a gauge-invariant matrix element $M^{(+)} = M_1^{(+)} + M_2^{(+)}$ is given by

$$M_2^{(+)} = -e f_{\rho\pi\pi} \epsilon^{(\rho)} \cdot \epsilon^{(\gamma)}. \quad (22)$$

In writing (21) we assume that the charged ρ meson has no anomalous magnetic moment and, accordingly, no electric quadrupole moment. The factor entering at the three-leg vertex with an incoming (p_ν) and outgoing (p_μ') vector meson coupled to a photon $\epsilon_\sigma^{(\gamma)}$ is[23,24]

$$(p+p')_\sigma \delta_{\mu\nu} - p_\nu' \delta_{\mu\sigma} - p_\mu \delta_{\nu\sigma}. \quad (23)$$

If it turns out that an anomalous magnetic moment plays a role in the electromagnetic interactions of the ρ meson, the prescriptions of reference 24 for the interaction between the electromagnetic field and charged vector mesons with arbitrary magnetic moment should be used to perform the calculations. We have used for the ρ propagator

$$D_{\mu\nu}^{(\rho)} = \left(g_{\mu\nu} - \frac{p_\mu^{(\rho)} p_\nu^{(\rho)}}{m_\rho^2} \right) \frac{1}{p^{(\rho)2} - m_\rho^2},$$

neglecting its width.

Using the matrix element $M^{(+)}$ we find

$$\Gamma_{\rho^{+,-}}(\pi^{+,-}\pi^0\gamma) = \alpha (f_{\rho\pi\pi}^2/4\pi)(3\pi m_\rho)^{-1} \int_k^{k_m} J(k) \frac{dk}{k}, \quad (24)$$

where

$$J(k) = \left[\left(\frac{m_\rho}{2} - k \right) k_m + \frac{3m_\pi^2}{m_\rho} k \right] \ln \frac{1+\xi}{1-\xi} - \xi \left\{ 2 \left(\frac{m_\rho}{2} - k \right) \left[k + k_m + \frac{k\xi^2}{m_\rho} \left(\frac{m_\rho}{6} + \frac{k}{8} \right) \right] - \frac{k^2}{m_\rho} \left[\tfrac{1}{3}(k_m - k) + \frac{m_\rho}{2} \right] - \frac{k^2 \xi^2}{24} + \frac{2m_\pi^2 k}{m_\rho} \right\} \quad (25)$$

with ξ defined in (13).

The relative decay probability for emitting a photon with energy greater than k will be given by

$$R_k' = \frac{\Gamma_{\rho^{+,-}}(\pi^{+,-}\pi^0\gamma)}{\Gamma_{\rho^{+,-}}(\pi^{+,-}\pi^0)} = 4\alpha m_\rho (\pi)^{-1} (m_\rho^2 - 4m_\pi^2)^{-3/2} \times \int_k^{k_m} J(k) \frac{dk}{k} \quad (26)$$

which is plotted in Fig. 2, while a few values are given in Table I.

Now we proceed to estimate separately the direct decay $\bar{\Gamma}$, contributed by the $\omega\pi$ intermediate state [without including the interference with $M^{(+)}$]. The calculation is analogous to the one discussed in the previous section, and it proves convenient to work in the $(\pi^0+\gamma)$ c.m. system. We find

$$\bar{\Gamma}_{\rho^{+,-}}(\pi^{+,-}\pi^0\gamma) = (f_{\rho\omega\pi}^2/4\pi) \cdot (f_{\omega\pi\gamma}^2/4\pi) \times m_\pi^2 (36\pi m_\rho)^{-1} (1.25 \times 10^2). \quad (27)$$

For the effective $\omega\pi\gamma$ vertex we use

$$(f_{\omega\pi\gamma}/m_\pi) \cdot \epsilon_{\alpha\beta\gamma\delta} p_\alpha^{(\omega)} \epsilon_\beta^{(\omega)} k_\gamma \epsilon_\delta^{(\gamma)}$$

and we neglect, as before, the ω width in the expression for its propagator. If we relate the $f_{\omega\pi\gamma}$ effective coupling

[23] R. P. Feynman, Phys. Rev. **76**, 769 (1949).
[24] T. D. Lee and C. N. Yang, Phys. Rev. **128**, 885 (1962).

to the $f_{\rho\omega\pi}$ vertex one has $f_{\omega\pi\gamma}=ef_{\rho\omega\pi}/f_\rho$ and we get

$$\bar{\Gamma}_{\rho^{+,-}}(\pi^{+,-}\pi^0\gamma)=\alpha(f_{\rho\omega\pi}^2/4\pi)^2(f_\rho^2/4\pi)^{-1}\times m_\pi^2(36\pi m_\rho)^{-1}(1.25\times10^2). \quad (28)$$

The ratio $\bar{\Gamma}_{\rho^{+,-}}(\pi^{+,-}\pi^0\gamma)/\Gamma_{\rho^{+,-}}(\pi^{+,-}\pi^0)$ is given in column 4 of Table II. A few remarks should be made at this point. The values of $\bar{\Gamma}_{\rho^{+,-}}(\pi^{+,-}\pi^0\gamma)$ and $\Gamma_{\rho^0}(\pi^0\pi^0\gamma)$, for the same $f_{\rho\omega\pi}$, are quite different even after allowance is made for a factor of $\frac{1}{2}$ appearing in the latter because of the indistinguishability of the neutral pions. The reason for it is that in the former decay magnetic transitions which bring P state pion pairs in the final state take place, in addition to the electric transitions which are the only ones contributing to the ρ^0 decay. The structure of the $\rho\rightarrow 2\pi+\gamma$ matrix element, due to the expression for the $\rho\omega\pi$ vertex, favors the appearance of the higher pion relative momenta in the final state, increasing the partial width for the decay of the charged ρ. The interference term which appears in the $\rho^0\rightarrow 2\pi^0+\gamma$ decay is responsible for the reduction in its rate.

Now we compare $\bar{\Gamma}_{\rho^{+,-}}(\pi^{+,-}\pi^0\gamma)/\Gamma_{\rho^{+,-}}(\pi^{+,-}\pi^0)$ given for different values of $f_{\rho\omega\pi}^2/4\pi$ in Table II with the values of R_k' (Table I) taken for, say, $k>30$ MeV. One sees immediately that only for small values of $f_{\rho\omega\pi}^2/4\pi$ (<0.1) we are allowed to neglect the interference between the two types of decay leading to $\pi^{+,-}\pi^0\gamma$. The very different γ-energy spectra of these two contributions makes it, however, difficult to estimate the neglected term. It should be reminded that the γ-energy spectrum obtained from the charged decay through the $\rho\omega\pi$ vertex is not identical to the one from the decay of the neutral ρ through the same vertex. The former has a more symmetric appearance because, with the inclusion of the odd angular momentum waves for the pion system, the probability for lower energy photons is increased.

5. SUMMARY AND DISCUSSION

The first-order electromagnetic decays of the ρ meson, namely, $\rho\rightarrow\pi+\gamma$, $\rho\rightarrow\eta+\gamma$, and $\rho\rightarrow 2\pi+\gamma$ have been estimated in this article with the aid of a detailed mechanism. The philosophy of the underlying model is the dominance of the low-mass intermediate states in the decay vertices, in a dispersion-theory sense. We have included in this work the 2π and 4π intermediate states. The existence of the three-pion isoscalar resonance, the ω meson, suggests the treatment of the 4π relevant state as an $\omega\pi$ system. This leads to a unified description of the strong and pionic radiative decays of ω and ρ using the $\rho\pi\pi$ and $\rho\omega\pi$ vertices only, much in line with the concepts of unitary symmetry.

The $(2\pi+\gamma)$ decay of the charged and neutral ρ mesons is found to be at least of about the same strength as the $(\pi+\gamma)$ one. The still unknown parameter of the model is the value of the $\rho\omega\pi$ coupling constant. If this coupling is of the order of magnitude as estimated from the π^0 decay,[6,7] and $\rho\rightarrow\pi+\gamma$ is indeed proceeding as described above, then the $2\pi+\gamma$ mode is the predominant electromagnetic decay of the ρ meson.

The $\rho\rightarrow\eta+\gamma$ mode, using unitary symmetry, is about one-third of the $\rho\rightarrow\pi+\gamma$ decay mode.

Within our approach, there is no contribution from the 4π intermediate state to the $\rho^0\rightarrow\pi^++\pi^-+\gamma$ transition. In the $\rho^{+,-}\rightarrow\pi^{+,-}+\pi^0+\gamma$ decay, however, the contributions of the "internal bremsstrahlung" (with infrared divergence cut off at 15-MeV photon energy) and of the "direct" part due to the 4π intermediate state are comparable for certain values of $f_{\rho\omega\pi}$.

The measurement of the processes estimated in this work, and the comparison of their widths to those of the ω decays will provide a decisive test of our working hypothesis, the dominance of the mesonic light intermediate states in the decays of the "new" mesons. One should not expect more than approximate agreement, as the inclusion of the neglected nucleonic states could induce some changes in the rates. Moreover, even for the mesonic vertices used we have no knowledge of their actual off-the-mass-shell behavior, and in addition we have used the strict equality $f_\omega=f_\rho=f_{\rho\pi\pi}$.

At present, no direct experimental measurement of any of the calculated electromagnetic decays of ρ exists. Preliminary results[25] on the photoproduction of ρ mesons indicate a value of 0.6% for $\Gamma_{\rho^0}(\pi^0\gamma)/\Gamma_{\rho^0}(\pi^+\pi^-)$. This would imply for the $f_{\rho\omega\pi}^2/4\pi$ coupling constant, assuming the $\rho\rightarrow\pi+\gamma$ decay is proceeding through this vertex, a value of $\simeq0.6$. We should remark that for calculational convenience we used $f_{\rho\omega\pi}/m_\pi$ to define a nondimensional coupling constant, although a more "characteristic" mass for this vertex would be m_ρ or m_ω. This means that the "true" value of the coupling constant is about 18, indicating a very strong coupling.

Recently, evidence has been presented for an enhancement at about 1020 MeV in the effective-mass distribution for the $K\bar{K}$ system. Bertanza *et al.*[26] have studied the Kpairs $\bar{K}$ from the reaction $K^-+p\rightarrow\Lambda+K+\bar{K}$ at 2.24 and 2.5 BeV/c and the final states which exhibit this behavior are apparently of the $K_1^0K_2^0$ type. Alexander *et al.*[27] have used the reaction $\pi^-+p\rightarrow K+\bar{K}+N$ at momenta ranging from 1.51 to 2.25 BeV/c and have found an enhancement in $K_1^0K_1^0$ final states at approximately the same energy as in reference 26.

It has been suggested[28] that the peak in the $K_1^0K_2^0$

[25] This value for the $\rho\pi\gamma$ effective coupling constant is obtainable by assuming the pion exchange to be the dominant diagram in the process $\gamma+p\rightarrow\rho+p$ observed by D. McLeod, S. Richert, and A. Silverman, Phys. Rev. Letters **7**, 383 (1961); S. B. Berman, in *Proceedings of the Conference on Photon Interactions* [Technology Press, Cambridge, Massachusetts, 1963 (to be published)]. See also, W. Alles and D. Boccaletti, Nuovo Cimento **27**, 306 (1963).

[26] L. Bertanza, V. Brisson, P. L. Connolly, E. L. Hart, I. S. Mitra, G. C. Moneti, R. R. Rau, N. P. Samios, I. O. Skillicorn, S. S. Yamamato, M. Goldberg, L. Gray, J. Leitner, S. Lichtman, and J. Westgard, Phys. Rev. Letters **9**, 180 (1962).

[27] G. Alexander, O. I. Dahl, L. Jacobs, G. R. Kalbfleisch, D. H. Miller, A. Rittenberg, J. Schwartz, and G. A. Smith, Phys. Rev. Letters **9**, 460 (1962).

[28] J. J. Sakurai, Phys. Rev. Letters **9**, 472 (1962).

mass distribution is indicative of the existence of another $T=0$ vector meson, called φ. If this proves to be correct, the observed states ω (780 MeV) and φ (1020 MeV) may well be mixtures of a $T=0$ vector meson belonging to a unitary octet with another $T=0$ vector meson which represents a unitary singlet. In this case, our estimates for the decay rates and the relations between the different partial widths connected with the $\rho\omega\pi$ vertex, will be affected. A quantitative analysis of this effect at the present stage of experimental knowledge seems to be premature.

Finally, we add a remark on the other strong decay of ρ, namely, $\rho\rightarrow 4\pi$. It could well be that this transition proceeds mainly through the $\omega\rho\pi$ vertex, i.e., $\rho^{+,-,0}\rightarrow(\omega)+\pi^{+,-,0}\rightarrow(\pi^{+}+\pi^{0}+\pi^{-})+\pi^{+,-,0}$. This would imply that the $\pi^{+}+\pi^{-}+2\pi^{0}$ configuration is prevalent in the $\rho^{0}\rightarrow 4\pi$ decay. The knowledge of $f_{\rho\omega\pi}$ would allow us to estimate the partial width of the decay. A reported experimental limit is[29]

$$(\rho^{+}\rightarrow\pi^{+}+\pi^{0}+\pi^{-}+\pi^{+})/(\rho^{+}\rightarrow\pi^{+}+\pi^{0})<5\%.$$

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[29] N. H. Xuong and G. R. Lynch, Phys. Rev. **128**, 1849 (1962)

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Energy Dependence of Proton Electromagnetic Form Factors

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A χ^2 analysis of electron-proton scattering is made to test for the energy dependence of the form factors, in the Regge pole form, using the slope of the pole as a parameter. It is found that the form factors can tolerate considerable energy dependence. However, decisive conclusion regarding such an energy dependence has to await the data at higher energies.

In the light of the discussion of the photon as a Regge pole by Blankenbecler, Cook, and Goldberger[1] a reanalysis of the proton-electron scattering data is made here.

The cross section for the p-e scattering via a single photon exchange is given in terms of the charge form factor $F_1(q^2)$ and momentum form factor $F_2(q^2)$ by the Rosenbluth formula:

$$\frac{d\sigma}{d\Omega}=\frac{e^2}{4(4\pi)^2E_0{}^2}\frac{\cos^2(\theta/2)}{\sin^4(\theta/2)}\frac{1}{1+(2E_0/M)\sin^2(\theta/2)}\times\left\{F_1{}^2-\frac{q^2}{4M^2}[2(F_1+2MF_2)^2\tan^2(\theta/2)+(2MF_2)^2]\right\},$$

where

$$q^2=-\{[2E_0\sin(\theta/2)]^2/[1+(2E_0/M)\sin^2(\theta/2)]\},$$

E_0= incident energy and θ= scattering angle of the electron in the laboratory system.

Fifty-three pieces of data for the cross section and probable errors are taken from Bumiller, Croissiaux, Dally, and Hofstadter,[2] out of a total of fifty-eight pieces, five being discarded on the basis that they do not lie on a smooth curve. The discarded pieces are

	Energy (MeV)	Angle (degree)
1.	700	135
2.	700	145
3.	850	135
4.	850	145
5.	900	75

The two-pole formula for the two form factors which has been found adequate for p-e and n-e scattering data by Hofstadter, de Vries, and Herman[3] was tried first and χ^2 was calculated.

$$F_1=e[1+A_1q^2/(q^2+A_5)+A_2q^2/(q^2+A_6)],$$
$$F_2=1.79(e/2M)[1+A_3q^2/(q^2+A_5)+A_4q^2/(q^2+A_6)],$$

where M= nucleon mass.

$$\chi^2=\sum[(d\sigma/d\Omega|_{\rm exp}-(d\sigma/d\Omega|_{\rm calc})/\Delta(d\sigma/d\Omega)]^2,$$

where $\Delta(d\sigma/d\Omega)$= standard deviation.

The fit is considered to be good if $\chi^2\leq N-n$, where N= total number of pieces of data $=53$ in the present case, n= number of parameters used.

The values of parameters A_i $(i=1,6)$ given by Hof-

[1] R. Blankenbecler, L. F. Cook, and M. L. Goldberger, Phys. Rev. Letters **8**, 463 (1962).
[2] F. Bumiller, M. Croissaux, E. Dally, and R. Hofstadter, Phys. Rev. **124**, 1623 (1961).
[3] C. de Vries, R. Hofstadter, and Robert Herman, Phys. Rev. Letters **8**, 381 (1962).